DETECTION OF LINES IN IMAGES

BACKGROUND OF THE INVENTION

1. Field of the Invention

This invention relates to the field of imaging, the field of image data, the field of analyzing image data, the field of analyzing digital image data, and the detection of lines and line-type features in images through analysis of image data.

2. Background of the Art

In digital image processing, there is frequently a need to detect lines in an image. The range of applications in which such a need arises is very broad. Examples of the situations in which detection may be particularly important include vectorization of raster engineering drawings, detection of structures such as nerves or blood vessels in medical images, identification of roads in satellite images or maps, lane line detection for autonomous vehicles, crack detection, defect detection in images, detection of weather features, detection of boundaries, and handwriting recognition, to name but a few areas of practice where line detection can be particularly important.

In the most general sense, line detection involves recognition of curvilinear structures, not just straight lines. Lines according to the present invention are therefore defined as a collection of contiguous or continuously associated points with the association having one predominant dimension (e.g., length) in the collection of points that is much larger (e.g., greater than 5:1, greater than 10:1, greater than 15:1, greater than 25:1, greater than 50:1, greater than 100:1, etc.) than another dimension (e.g., width), with the ratio being dependent, in part, upon the absolute size of the major dimension. For example, a line of 100 millimeters in length and 0.2 millimeters in width caused by a razor cut across an image surface is no less a line if the cut is only 2 millimeters in length and 0.2 millimeters in width) than the other dimension (width). The

dimension of depth in the actual article is generally immaterial, as depth will not be a factor in two-dimensional image data that can be used to generate an image, although image features relating to depth, such as shadowing, roughness, etc. may be included within the image data. Such line structures are distinct from object boundaries, which are demarcations between large areas of different brightness, even though both boundaries and lines in an image exhibit an edge response. While a line of finite width has edges, it is characterized by being much longer than it is wide (as described above) and is thus distinct from the edges created at the boundaries of extended objects in an image. Detection of a line requires that both its location and width can be accurately described, since frequently, only lines within a certain range of widths are of interest.

Most of the approaches to line detection are based in some way on detection of edges. Methods of edge detection have been reviewed by D. Ziou and S. Tabbone, "Edge Detection Techniques", *Technical Report No. 195*, Département de Mathématiques et Informatique, Université de Sherbrooke, Canada, 1997. Schemes for line detection range from a concentration on local brightness differences in the image through detection of lines as objects having parallel edges to more sophisticated techniques. Some of these more sophisticated techniques use the curvature of the brightness of the image for estimation of lines using contours, ridges and ravines and locally fitting the curvature of the image. The field of line detection has been reviewed by C. Steger, "An Unbiased Detector of Curvilinear Structures", *Technical Report FGBV-96-03*, Forschungsgruppe Bildverstehen, Informatik IX, Technische Universität München, Germany, 1996. Many methods are of considerable computational complexity and there remains a need for simple and rapid methods for detecting lines of different scales in the presence of other image features.

In M. I. Trifonov and P. A. Medinnikov, *Sov. J. Opt. Technol.*, **58**, 235-238 (1991) there is reported a local radial angular transform for images ('loral' transform). This transform is described for a hexagonal grid representation of a binary image, which does not conform to the conventional representation of an image as pixels on a rectangular grid. The same transform in a

hierarchical hexagonal form was also briefly discussed in M. I. Trifonov and Yu. E. Shelepin, *Perception*, **21, Suppl. 2**, 54 (1992). In this publication, the role of the transform as a possible model for the early stages of human vision was considered with relation to the ability for visual detection of shapes. However, no practical application of the local radial-angular transform in image processing has been reported.

SUMMARY OF THE INVENTION

One aspect of this invention comprises methods of image processing based on a local radial angular transform. Another aspect of the invention is to provide a method of detecting lines and line-like features in images using the local radial angular transform. Another aspect of the invention is to provide a means of detection of lines in images in the presence of edges caused by the boundaries of other objects, distinguishing between the two features. A still further aspect of the invention is to provide a means of detecting lines of predetermined width in an image by means of the local radial angular transform. Yet another aspect of the invention is to provide a means of hierarchical description of the lines in an image according to scale by means of a local radial angular transform. An additional aspect of the invention is to provide a means of detecting other image features or shapes, including semi-planes, triangles, line junctions, rings or disks.

BRIEF DESCRIPTION OF THE FIGURES

Figure 1 shows a hexon with quasipixels formed as squares of size 2 by 2 in number of actual image pixels.

Figure 2 shows two orientations of a hexon based on 2 pixel by 2 pixel quasipixels that are not contiguous. This illustrates two orientations based on the same quasipixel with a larger spacing

Figure 3 illustrates two orientations in a hexon based on the same quasipixel of Figure 2 with a larger spacing.

Figure 4 illustrates two orientations in a hexon based on the same quasipixel of Figure 2 with still larger spacing.

Figure 5 illustrates two orientations in a hexon based on the same quasipixel of Figure 2 with still larger spacing.

Figure 6 illustrates an orientation of a quasipixel that does not fall exactly on pixels boundaries, illustrating a situation where pixels have partial membership of a given quasipixel

Figure 7 shows hexons in a first additional arrangement based on quasipixels having a size of one pixel.

Figure 8 shows hexons in a second additional arrangement based on quasipixels having a size of one pixel.

Figure 9 shows hexons formed from contiguous quasipixels that are 3 pixels by 3 pixels in size.

Figure 10 shows hexons formed from non-contiguous quasipixels that are 3 pixels by 3 pixels in size.

Figure 11 shows hexons formed from non-contiguous quasipixels that are 3 pixels by 3 pixels in size.

Figure 12 shows a first arrangement of quasipixels based on 4 by 4 pixel quasipixels.

Figure 13 shows a second arrangement of quasipixels based on 4 by 4 pixel quasipixels.

Figure 14 shows a third arrangement of quasipixels based on 4 by 4 pixel quasipixel.

Figure 15 shows quasipixels with contiguity composed of five image pixels or by virtue of corner-to-corner contact.

Figure 16 shows quasipixels with contiguity comprising four image pixels.

Figure 17 shows quasipixels where hexons are organized hierarchically.

Figure 18 shows the detection of straight lines of various widths and orientations (at the top of the Figure), an image of $|c_3|$ responses of the local radial angular transform obtained as $\max(|c_{13}|, |c_{23}|)$ using the hexon of Figure 1 (in the center of the Figure 18) under the constraint $\max(\delta_{13}, \delta_{23})\ 0.49$.

Figure 19 shows a comparison of detected original line-containing figures, and line detections by LORA analysis and Sobel analysis.

Figure 20 shows a comparison of original line-containing figures, and line detections by LORA analysis and Sobel analysis.

Figure 21 shows a comparison of original line-containing figures, and line detections by LORA analysis and Sobel analysis.

Figure 22 shows the use lines of width 2, 6 and 12 pixels and that a particular line width can be detected in the presence of lines of other widths.

Figure 23 shows a line that increases progressively from a width of one pixel to a width of 20 pixels.

Figure 24 shows a line – 4 pixels at its narrowest – whose edges become progressively more blurred and indistinct along its length.

Figure 25 shows dashed lines that are magnified two-fold to show detail. The upper portion shows a line of width 6 pixels, with a dash length of 30 pixels and a gap length of 20 pixels.

Figure 26 shows a collection of filled shapes, empty shapes and lines. The lines are 2 pixels wide. At bottom is shown the response of the Sobel edge filter to this image. All the objects are detected, with lines appearing as double edges. The center shows the LORA $|c_2|$ response of the hexon of Figure 12.

Figure 27 shows a variety of black shapes on a 50% gray background. This image was processed with the hexon of Example 5 having 6 pixel by 6 pixel quasipixels and with $\max(\delta_{14}, \delta_{24})$ 0.81.

Figure 28 shapes on a 50% gray background, magnified six-fold for clarity. The image was processed with the hexon of Example 5 having 10 pixel by 10 pixel quasipixels with restriction of responses.

DETAILED DESCRIPTION OF THE INVENTION

A digital image comprises a collection of picture elements or pixels arranged on a regular (e.g., Cartesian-type, or other point-by-point identifiable grid, pixel-by-pixel identifiable grid, with point or pixel identification indicating a specific space within a field) grid. A gray scale image is typically represented by a channel of specific brightness values at individual pixel locations. Such a channel may also be represented as a color palette, for example, a palette containing 256 shades of gray. A color image contains several channels, usually three or four channels, to describe the color at a pixel. For example, there may be red, green and blue (RGB) channels, or cyan, magenta, yellow channels (CYM) and black as the fourth (CMYK) channel. Each channel again contains brightness values representing the color at each pixel. Additionally, the image may be represented in a color space having a lightness channel along with other channels directly or indirectly representing the hue and saturation components of color. Examples include the HLS, HSV, YIQ, YUV, YES, CIE L*u*v* and CIE L*a*b* color spaces. All the aforementioned channels and any other image data formats for describing color space that include a brightness component are particularly suitable for the practice of the invention, although other color image data formats may be used with less facility.

The method of this invention may use a local radial angular transform (referred to herein as a 'lora' or 'LORA' transform) utilizing a hexagonal structuring of pixels or groups of pixels, termed a hexon, overlaid over the pixels of the image. Any hexagonal structuring of pixels or groups of pixels with reference to a central pixel, especially those that approach geometric symmetry or are close to the level of geometric symmetry allowed by the use of pixels (in one of or both a horizontal direction and a vertical direction) may be used as the hexagonal geometric structure. The description of the invention, for brevity, will focus on an illustrated group of pixels as representative of hexons (a distribution of six pixels or six groups of pixels surrounding a central pixel or central group of pixels). Because the hexons are usually selected with whole integer Cartesian coordinates as their centers, there are few, if any, geometric structures that will be provided as regular polygons, even though they may have two different symmetries (vertical

and horizontal, although the symmetries are not identical as they would be with regular polygons). The fact that the hexons are not necessarily regular is of no consequence to the practice of the invention. Procedures for performing this overlay will be described later, after the properties of the hexon have been described. The hexon consists of a central reference pixel or group of pixels surrounded by six pixels or six groups of pixels arranged in approximate two or three-way symmetry about the central group. The distribution of the groups of pixels about the central pixel may not or need not be truly symmetrical as is the case with a regular hexagon, as long as the distribution of the surrounding pixels remains approximately uniform when comparing different central pixels.

Figure 1 shows two non-limiting orientations of such a symmetrical, but non-regular hexagonal arrangement of pixel groups. The numbers shown in the figure identify each of the pixel groups and appear as subscript labels in the following description.

For any channel of the image, the average brightness values or channel values can be computed within any one of the peripheral groups of pixels surrounding the central reference group. It is preferred that the groups are chosen in such a way that, if the image is everywhere of exactly the same arbitrary non-zero brightness, the sum of the brightness values of all the pixels in every group is a constant. That is to say, it is preferred that, while the groups may be of any shape, size or orientation, even different from each other, if each such group were to be placed in turn over the same region of an image the sum of brightness values in each group would be approximately the same. This sum may be computed as a simple sum or a weighted sum. In an image having any particular brightness values at its pixels, the mean brightness values of the six groups surrounding the central group may be represented as a vector $B = (B_1, B_2, B_3, B_4, B_5, B_6)^T$, where the superscript T denotes a transpose operation that converts a row matrix to a column matrix. A Local Radial Angular (LORA) transform L c is defined as $c = RB$, where $c = (c_1, c_2, c_3, c_4, c_5, c_6)^T$ is a vector of transformation coefficients. R in the case of the hexon is a six by six square matrix whose elements are formed according to:

$$R_{km} = (1/6)\ exp[i\,(k-1)\,(m-1)\,\pi/3]\ ,\ (k, m = 1,2 \ldots 6)$$

where i is the imaginary unity (i.e., the square root of -1), π is the ratio of the circumference to the diameter of a circle, and k and m are the row and column indices of the matrix elements. For another geometric figure, the numbers would be adjusted to reflect the different number of groups (e.g., 8, 10 12, etc., in the geometric pattern). The matrix R may be represented for convenience in terms of a real matrix P and an imaginary matrix Q according to:

$$R = [1/(26)]\ P + i\ [1/(22)]\ Q$$

The elements of the matrix P are formed according to:

$$P_{km} = 2\ cos[\ (k-1)\,(m-1)\,\pi/3]\ ,\ (k, m = 1,2 \ldots 6)$$

and those of Q according to:

$$Q_{km} = (2/3)\ sin[\ (k-1)\,(m-1)\,\pi/3]\ ,\ (k, m = 1,2 \ldots 6)$$

P and Q are integer matrices with the explicit form shown below.

$$P = \begin{vmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 1 & -1 & -2 & -1 & 1 \\ 2 & -1 & -1 & 2 & -1 & -1 \\ 2 & -2 & 2 & -2 & 2 & -2 \\ 2 & -1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -1 & -2 & -1 & 1 \end{vmatrix} \qquad Q = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & -1 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 \\ 0 & -1 & -1 & 0 & 1 & 1 \end{vmatrix}$$

Any one of the six transformation coefficients (k = 1,2 … 6) can be computed according to:

$$c_k = R_{k1}B_1 + R_{k2}B_2 + R_{k3}B_3 + R_{k4}B_4 + R_{k5}B_5 + R_{k6}B_6$$

Through the use of matrices P and Q, the real and imaginary parts of any of the six transformation coefficients (k=1,2 … 6) can be computed independently, thus:

$$\mathrm{Real}(c_k) = (0.5/6)\,(P_{k1}B_1 + P_{k2}B_2 + P_{k3}B_3 + P_{k4}B_4 + P_{k5}B_5 + P_{k6}B_6)$$
$$\mathrm{Imaginary}(c_k) = (0.5/2)\,(Q_{k1}B_1 + Q_{k2}B_2 + Q_{k3}B_3 + Q_{k4}B_4 + Q_{k5}B_5 + Q_{k6}B_6)$$

The resulting expressions for each of the transformation coefficients are given below.

$$c_1 = (1/6)\,(B_1 + B_2 + B_3 + B_4 + B_5 + B_6)$$
$$c_2 = (0.5/6)\,(2B_1 + B_2 - B_3 - 2B_4 - B_5 + B_6) + i\,(0.5/2)\,(B_2 + B_3 - B_5 - B_6)$$
$$c_3 = (0.5/6)\,(2B_1 - B_2 - B_3 + 2B_4 - B_5 - B_6) + i\,(0.5/2)\,(B_2 - B_3 + B_5 - B_6)$$
$$c_4 = (1/6)\,(B_1 - B_2 + B_3 - B_4 + B_5 - B_6)$$
$$c_5 = (0.5/6)\,(2B_1 - B_2 - B_3 + 2B_4 - B_5 - B_6) - i\,(0.5/2)\,(B_2 - B_3 + B_5 - B_6)$$
$$c_6 = (0.5/6)\,(2B_1 + B_2 - B_3 - 2B_4 - B_5 + B_6) - i\,(0.5/2)\,(B_2 + B_3 - B_5 - B_6)$$

It can be seen that c6 is the complex conjugate of c2, and c5 is the complex conjugate of c3. Since the modulus $|z|$ of a complex number $z = a + ib$ is given by $|z| = (a^2 + b^2)$, then $|c_2| = |c_6|$ and $|c_3| = |c_5|$. Further, it can be noted that coefficient c_1 is simply indicative of the mean channel value or brightness under the hexon. There are separate coefficients c_{1k} and c_{2k} for the orientations 1 and 2 of a hexon, such as shown in Figure 1.

The modulus $|c_3|$ of the transformation coefficient c_3 has been found to be a significant quantity. The magnitude of this modulus is an indicator of the presence of a line-like feature in the image lying under the hexon superimposed over the image. Referring to Figure 1, for orientation 1 of the hexon, the orientation θ_1 of the line-like feature measured anticlockwise from the image horizontal is:

$$\theta_1 = \mathit{arctan}[\mathrm{Imaginary}(c_3) / \mathrm{Real}(c_3)]$$

while for orientation 2 or the hexon, the orientation θ_2 of the line-like feature is:

$$\theta_2 = \mathit{arctan}[\mathrm{Imaginary}(c_3) / \mathrm{Real}(c_3)] + \pi/2$$

The angle θ is a continuous function of c_3, giving rise to angles in the interval 0 to 180 degrees, but for exact hexagonal symmetry, the highest accuracy in θ is obtained near (but not necessarily exactly at) angles of 0, 30, 60, 90, 120 and 150 degrees. In practice, the θ angles of highest accuracy deviate slightly from these values because the requirement to map the hexon to pixels on a square grid leads to slight distortions of the hexagonal symmetry. For example, in the arrangement shown in Figure 1, the angles of highest accuracy in θ are 0, 26.6, 63.4, 90, 116.6 and 153.4 degrees. The estimate of the orientation angle of the line-like feature can also, for instance, be improved by computing the angle as an average of θ_1 and θ_2 weighted by the $|c_3|$ responses of the two orientations of the hexon.

As noted earlier, the hexon may be composed of groups of pixels, termed quasipixels for convenience or nomenclature. The hexon allowed is quite broad in scope, even though certain specific pixel and pixel group distributions are preferred. By way of non-limiting examples, the following arrangement of pixels, in addition to those shown in the Figures and elsewhere described in the specification, are provided:

Quasipixel 1 = 1 pixel

Quasipixel 2 = a group of 49 pixels in a 7 x 7 pixel square whose output is scaled by dividing by 49.

Quasipixel 3 = 49 pixels arranged in a vertical row whose output is scaled by dividing by 49.

Quasipixel 4 = 5 pixels arranged in an L-shape whose output is scaled by dividing by 5.

Quasipixel 5 = a hollow triangle of 12 pixels whose output is scaled by dividing by 12.

Quasipixel 6 = 5 pixels arranged in a cross with the center pixel receiving twice the weight of other pixels and whose output is scaled by dividing by 6.

Pixel groups are distributed about the central pixel or central group of pixels, preferably with some form of symmetry, but as noted elsewhere, the hexon can be weighted or eccentrically located about the central pixel. In Figure 1, the quasipixels are formed as squares of size 2 by 2 in number of actual image pixels. In this example, the quasipixels are shown in this Figure 1 as contiguous. The invention, however, envisages a very broad range of arrangements of quasipixels, distributions of pixels within the quasipixels, numbers of pixels within the quasipixels, and the like. The size, shape, pixel membership and spacing of the quasipixel groups can vary widely, provided that approximate hexagonal distributions, (including, for example, symmetry) are maintained. As the quasipixel size is varied, line features of different width or scale may be detected selectively. When the quasipixels are spaced apart, it becomes possible to detect dashed lines. Some illustrative, but not restrictive examples are shown in the figures. Figure 2 demonstrates two orientations of a hexon based on 2 pixel by 2 pixel quasipixels that are not contiguous. Figure 3 illustrates two orientations based on the same quasipixel with a larger spacing. Figures 4 and 5 show larger spacings still. In Figure 6 the quasipixel does not fall exactly on pixels boundaries, illustrating an acceptable situation according to the invention where pixels have partial membership of a given quasipixel. In the figure, individual image pixels are designated by lower case letters and the quasipixels are designated by numbers. Thus, for example, quasipixel 5 has the pixels a and b as partial members. The mean brightness at quasipixel 5, B_5, can be calculated as a weighted sum of the contributions B_a and B_b of the pixels a and b so:

$$B_5 = 0.5\ B_a + 0.5\ B_b$$

This permits the hexon to achieve pixel-level resolution. The values of B_2, B_3 and B_6 are computed analogously, while B_1 and B_4 are obtained directly from the brightness of the underlying pixels f and d respectively. Figures 7 and 8 indicate additional arrangements based on quasipixels having a size of one pixel. For the rightmost variant in Figure 8, quasipixels 2, 3, 5

and 6 again have partial pixel membership. Hexons formed from quasipixels that are 3 pixels by 3 pixels in size are illustrated in Figures 9, 10 and 11. The quasipixels are contiguous in Figure 9 and discontiguous in Figures 10 and 11. Similar arrangements based on 4 by 4 pixel quasipixels are shown in Figures 12 to 14. The choice of hexons composed of quasipixels of a different size allows line features of different widths to be detected selectively. In all the examples, the pixels forming the quasipixels are part of the same group because of contact or contiguity of the individual pixels. This contiguity may be by virtue of side-to-side pixel contact as shown in Figure 15 for quasipixels composed of five image pixels or by virtue of corner-to-corner contact, which is shown in Figure 16 for quasipixels comprising four image pixels. Hexons may also be organized hierarchically as demonstrated in Figure 17. The large scale hexon comprises quasipixels that are themselves hexons constructed from smaller quasipixels corresponding to a size of one image pixel. Such an arrangement may be used for simultaneously characterizing texture in an image at several scales. Additionally, it is possible to successively process an image with a range of hexon sizes based on quasipixels of increasing size in order to achieve a multiscale description of image content quantized to a chosen scale interval. This description may be organized in hierarchical form. By virtue of the integer form of matrices P and Q processing is rendered very rapid compared to conventional methods of multiscale description such as those using Gabor filters.

The central reference pixel or group of pixels does not participate directly in the LORA transform. However, it represents the location of the image at which the results of the LORA transform may be considered to apply. In one view, the central pixel or group of pixels is a conceptual location relative to which the symmetry of the hexon may be defined. In another view, the central pixel or group of pixels is a location with which the various outputs of the LORA transform may be associated. For example, if it is desired to record one of the possible useful outputs of the LORA transform (e.g., a coefficient c_k, as described below) this may be done, for instance, by scanning the location of the central pixel group pixel-by-pixel across an input image and at each point assigning an output of the LORA transform to the equivalent

location of an output image. Other than using the reference central group as a conceptual center of the hexon (e.g., for defining symmetry or assigning an output value), it is not required to use the central reference group. Despite this, it is contemplated that pixel values in the central reference group may also be used in conjunction with pixel values of the six peripheral groups that are used to compute the LORA transform to provide additional useful information that may not be available from the LORA transform alone. If the reference central group is used in this way, it may also be considered as a quasipixel and it is preferred that the central group have the properties specified earlier for quasipixels.

For any hexon type, such as those exemplified in Figures 1 to 17, a coefficient c_k is calculated for every position of the hexon in relation to the image pixels. That is to say, the hexon center position is moved pixel by pixel across the rows and down the columns of the image until it has been positioned over every image pixel of interest. The result is an image formed from c_k responses that is identical in size to the original image area of interest. It is also possible to move the hexon in larger steps. For instance, hexons composed of quasipixels comprising several pixels may be moved in steps equal to the quasipixel width or height. In such a case the output image will be smaller than the input image, being effectively a lower resolution or coarser scale image. The value of c_k calculated for the hexon is a property of the central pixel of the hexon. In other words, it is a property of the central pixel of the quasipixel labeled 0 in Figures 1 to 17. When the width and height of the quasipixel is an odd number of pixels there is no ambiguity about the location of the center pixel. However, when the quasipixel has even dimensions, the center of the quasipixel does not coincide with an image pixel. In such a case it is usually sufficient to assign c_k to that image pixel lying closest to the center of the quasipixel. When several pixel lie at equal distances from the center of the quasipixel an arbitrary choice can be made. For example, referring to Figure 1, c_k may be assigned to the top left pixel of quasipixel 0. It is also possible in such a case to form an improved value of c_k at a given pixel by averaging the responses from several positions of the central quasipixel each lying over the pixel in question. For instance, referring to Figure 1, the response at a given pixel may also be formed as

the average of c_k values obtained by positioning in turn the top left, top right, bottom left and bottom right elements of quasipixel 0 over the given image pixel. Alternatively, all pixels lying under central quasipixel may be assigned the value of c_k determined for one pixel in this region. There is a separate value of $|c_k|$ for orientation 1 and orientation 2 of the hexon. These separate values $|c_{1k}|$ and $|c_{2k}|$ may be combined into a single value $|c_k|$, for instance by taking the larger of the two.

As noted before, the magnitude of $|c_3|$ is a measure of the presence of a line-like feature under the hexon. An image formed from $|c_3|$ responses is, therefore, an image of the line-like elements of the original image. $|c_3|$ may be scaled linearly or non-linearly to any convenient representation as a channel value of an image. It may be thresholded by whatever selection best meets the needs of the application. For instance, values of $|c_k|$ may be retained only if they exceed a threshold or only if they lie within a range defined by a lower and upper threshold. The magnitude of $|c_3|$ can be considered the strength of a line response in the image. It is also possible to define a different hexon response, δ_3, which is a measure of the line purity. This quantity is defined as:

$$\delta_3 = 2\,|c_3|^2 \Big/ \sum_{k=2}^{k=6} |c_k|^2$$

Other definitions involving weighted functions of the coefficients c_k are also possible. Since coefficients other than c_3 respond to image features that are not lines, δ_3 is a measure of the degree to which the c_3 response represents a line. There are separate values, δ_{13} and δ_{23}, of this measure for hexon orientation 1 and orientation 2 respectively. These may be combined into a single value of δ_3 by any convenient means, for instance by using the larger value. Thus there are two means of line analysis - $|c_3|$, the line strength metric, and δ_3, the line purity metric. The strength metric responds, for instance, to the contrast of the line relative to its surroundings. High contrast leads to a high value of $|c_3|$ and low contrast to a low value. The purity metric responds, for instance, to how well defined the line is. Lines with sharp edges yield high values of δ_3, while lines with blurred, ill-defined edges give smaller values of δ_3. The two metrics may be used

independently or in combination. It is preferred to use both metrics in combination since this allows the maximum selectivity for a particular type of line. For instance, δ_3 may be tested against a threshold (even an arbitrary threshold) so that values of $|c_3|$ are discarded or set to zero if δ_3 lies below the threshold. It is also effective to use a lower and upper δ_3 threshold so that $|c_3|$ responses are retained only when the value of δ_3 lies within a certain range.

The c_3 coefficient responds to lines that are both dark and light with respect to the background upon which they lie and by default both types of lines are detected. However, it is also possible to selectively detect only light lines or only dark lines. This may be achieved in various ways. For example, the mean brightness or channel value at the quasipixels lying closest to the line may be compared to the value of $c_1/6$. Alternatively, the lightness or darkness of a line may be estimated from the real and imaginary parts of the c_3 coefficient by comparison to thresholds T_1 and T_2 according to the following logic:

$$\text{if } |\text{Imaginary}(c_3) / \text{Real}(c_3)| \quad T_1 \text{ and } \text{Real}(c_3) > T_2 \text{ then Light}$$
$$\text{if } |\text{Imaginary}(c_3) / \text{Real}(c_3)| > T_1 \text{ and } \text{Real}(c_3) < T_2 \text{ then Light}$$
$$\text{if } |\text{Imaginary}(c_3) / \text{Real}(c_3)| \quad T_1 \text{ and } \text{Real}(c_3) < T_2 \text{ then Dark}$$
$$\text{if } |\text{Imaginary}(c_3) / \text{Real}(c_3)| > T_1 \text{ and } \text{Real}(c_3) > T_2 \text{ then Dark}$$

While the value of T_1 depends on the detailed geometry of the hexon, a preferred value of the threshold T_1 is from greater than about 0 to less than about 0.57. An especially preferred value is about 0.07 to about 0.41, about 0.15 to 0.35, and with a most especially preferred value of about 0.3. The preferred value of T_2 is about 0. In this way either light or dark lines may be separately detected.

The invention will be further illustrated with the following, non-limiting examples. Example 1 (Figure 18) shows at the top of the Figure a selection of straight lines of various widths and orientations. In the center is an image of $|c_3|$ responses of the local radial angular transform obtained as $\max(|c_{13}|, |c_{23}|)$ using the hexon of Figure 1 under the constraint $\max(\delta_{13}, \delta_{23})\ \ 0.49$. At the bottom are the responses of the conventional Sobel edge detection filter. It can

be seen that the LORA detector identifies the centers of both dark and light lines having width one, two and four pixels while ignoring lines with a width of seven pixels, which are wider than the four-pixel quasipixel of the hexon. In contrast the Sobel detector identifies the borders of the lines no matter what the size.

Example 2 (Figure 19) was obtained using the detector of Example 1, and shows that the LORA detector is capable of tracing the center of a light or dark irregular curve. Once again the Sobel filter detects the line edges rather than its center. The image has been magnified two-fold to better show detail.

Example 3 (Figure 20) contains several light and dark regular and irregular shapes show at the top. Some consist only of an outline while others are filled shapes. With the LORA detector of Example 1, only the outlines are detected. However, the conventional Sobel filter cannot distinguish between lines and the boundaries of filled objects.

Example 4 (Figure 21) shows on the left lines of differing width (1, 2 and 4 pixels) with different brightness relative to the gray background (0%, 12%, 25%, 37%, 62%, 75% 87% and 100% gray). The image has been magnified two-fold to reveal detail. On the right is the response of the LORA detector of Example 1, illustrating detection of lines irrespective of contrast. The magnitude of the $\max(|c_{13}|, |c_{23}|)$ response is, however, sensitive to the contrast.

Example 5 (Figure 22) shows using lines of width 2, 6 and 12 pixels that a particular line width can be detected in the presence of lines of other widths. The image has been magnified two-fold to show detail. All LORA detectors used a threshold $\max(\delta_{13}, \delta_{23})$ 0.49. The 2-pixel lines were detected with the hexon of Figure 1. The 6-pixel lines were detected with a 6 pixel by 6 pixel quasipixel arranged as in Figure 1 and a threshold of $\max(|c_{13}|, |c_{23}|)$ 0.84 $|c_3|_{max}$., where $|c_3|_{max}$ is the largest c_3 response anywhere in the image for a particular hexon. The 12-pixel lines were detected with a 10 pixel by 10 pixel quasipixel arranged as in Figure 1 and a threshold of

$\max(|c_{13}|, |c_{23}|)$ 0.84 $|c_3|_{max}$. The detected lines may be organized hierarchically with respect to scale or width.

Example 6 (Figure 23), which has been magnified two-fold, shows a line that increases progressively from a width of one pixel to a width of 20 pixels. Using the hexons of Example 5 with $\max(\delta_{13}, \delta_{23})$ 0.49 and $\max(|c_{13}|, |c_{23}|)$ 0.9 $|c_3|_{max}$, the ability to extract line sections of different distinct width is demonstrated.

Example 7 (Figure 24), again magnified two-fold, shows a line – 4 pixels at its narrowest – whose edges become progressively more blurred and indistinct along its length. The relevant detector thresholds are noted in the figure. Using the hexon of 6 by 6 quasipixels from Example 5 regions of different line sharpness may be detected. A lower δ_3 threshold results in the line being detected as having uniform width, while a threshold on $|c_3|$ permits only the sharp section of the line to be detected.

Example 8 (Figure 25) concerns dashed lines and is magnified two-fold to show detail. The upper portion shows a line of width 6 pixels, with a dash length of 30 pixels and a gap length of 20 pixels. The dash line elements are detected using the 6 by 6 quasipixel hexon of Example 5 with $\max(\delta_{13}, \delta_{23})$ 0.49. The lower portion shows a line of width 6 pixels, with a 30 pixel long dash and a 3 pixel wide gap. The gap elements of the line are detected using the hexon of Figure 1 with $\max(\delta_{13}, \delta_{23})$ 0.49 and $\max(|c_{13}|, |c_{23}|)$ 0.7 $|c_3|_{max}$.

Example 9 (Figure 26) shows at the top of the Figure a collection of filled shapes, empty shapes and lines. The lines are 2 pixels wide. At bottom is shown the response of the Sobel edge filter to this image. All the objects are detected, with lines appearing as double edges. The center shows the LORA $|c_2|$ response of the hexon of Figure 12 with $\max(\delta_{12}, \delta_{22})$ 0.49 and $\max(|c_{12}|, |c_{22}|)$ 0.7 $|c_2|_{max}$ illustrating that object boundaries may be selectively detected in the presence of other edges. In this case δ_2 is defined as:

$$\delta_2 = 2\,|c_2|^2 \,/ \sum_{k=2}^{k=6} |c_k|^2$$

Example 10 (Figure 27) shows a variety of black shapes on a 50% gray background. This image was processed with the hexon of Example 5 having 6 pixel by 6 pixel quasipixels and with $\max(\delta_{14}, \delta_{24})$ 0.81, the results being magnified six-fold for clarity. Superimposed in white are the locations of the $|c_4|$ responses of the hexon, which indicate a capability to detect triangular structures and certain types of line junctions and intersections. In this case δ_4 is defined as:

$$\delta_4 = |c_4|^2 \,/ \sum_{k=2}^{k=6} |c_k|^2$$

Example 11 (Figure 28) presents some shapes on a 50% gray background, magnified six-fold for clarity. The image was processed with the hexon of Example 5 having 10 pixel by 10 pixel quasipixels with restriction of responses according to $|c_2| < 0.39|c_k|_{max}$, $|c_3| < 0.39|c_k|_{max}$, $|c_4| < 0.39|c_k|_{max}$ and $|B_0 - c_1/6| > 0.21B_{max}$, where $|c_k|_{max}$ is the largest value of $|c_k|$ anywhere in the image and B_{max} is the maximum brightness of the image. In this example both of these values are 255. When all of $|c_2|$, $|c_3|$, $|c_4|$, $|c_5|$ and $|c_6|$ are small and $|B_0 - c_1/6|$ is large, then $|B_0 - c_1/6|$ responds to a disk-like or ring-like forms. The locations of this response are shown in Figure 28 in white for black shapes and in black for white shapes centered in the shapes. The responses demonstrate that disks and rings can be detected. Two disks are too large to detect with the hexon used and two rings are too thin to detect with this hexon.